INTERACTION OF MAGNETOHYDRODYNAMIC WAVES WITH CONTACT AND VORTEX DISCONTINUITIES

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The following interactions of magnetohydrodynamic waves are considered: fast (S^+) and slow (S^-) shock waves, fast (R^+) and slow (R^-) expansion waves, with vortex (A) and contact (K) discontinuities. The medium is assumed to be ideally conducting. No restrictions whatever are imposed on the parameters of the medium. At the moment of interaction there is created a discontinuity which must be resolved into some combination of waves. The purpose of this paper is to determine the possible combination of waves.

Interactions of shock waves and of expansion waves with a contact surface in gasdynamics were investigated in [1] and [2].

Problems in the resolution of an arbitrary discontinuity, created in the interaction of magnetohydrodynamic waves with vortex and contact discontinuities, are particular cases of the general problem of the resolution of an arbitrary discontinuity in the magnetohydrodynamics of an ideally conducting medium [3]. The symbols used are the same as in [3]. Instead of the expression: "the line in the pH_y-plane which represents the relation between the quantities p and H_y in an S^+(S^-, R^+, R^-)-wave with magnetic field and pressure ahead of the wave equal to H_0 and p_0", we shall write for conciseness: "the line S^+(S^-, R^+, R^-) leaving the point (p_0, H_{y_0})"; instead of the expression: "the combination which corresponds to points (lines and regions) lying in the quadrant $\Delta u > 0$, $\Delta v > 0$ of the $\Delta u \Delta v$-plane, Fig. 1", we shall write: "the combinations lying in the quadrant $\Delta u > 0$, $\Delta v > 0$, Fig.1".

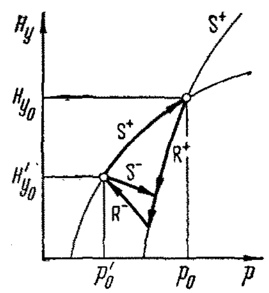

Fig. 1.

If R^+- and R^--waves do not participate in the interactions under consideration, then the waves which move apart in the two directions are created at the instant of impact. Interactions in which R^+- and R^--waves participate lead, as in gasdynamics [2], to a process of interpenetration of the waves during which the motion cannot be described by means of simple waves. In this case, if the interpenetration is completed in a finite time, as will be assumed in what follows, the combinations of waves emerging from the interpenetration zone must, generally speaking, be composed of shock waves, simple waves and vortex discontinuities. The possible wave combinations which are created in interactions with R^+- and R^--waves are determined in what follows, omitting the motion in the region of interpenetration.

For definiteness, we shall assume the following:

1) The x-axis is perpendicular to the plane of the interacting waves, and the direction of the y-axis coincides with the direction of **H** in the undisturbed medium.

2) In the interaction of magnetohydrodynamic waves with a K-discontinuity, the K-discontinuity is taken to be at rest, magnetohydrodynamic shock waves overtake it from the left, and expansion waves from the right.

3) A vortex discontinuity overtaken by fast magnetohydrodynamic waves, and slow waves overtaken by a vortex discontinuity, move to the right.

4) A vortex discontinuity colliding with magnetohydrodynamic waves moves to the left, and consequently the waves move to the right.

1. Interaction of an S^+-wave with K- and A-discontinuities. Let us consider the plane case of the resolution of an arbitrary discontinuity when the point corresponding to the condition (p_0, H_{y_0}) lies in the pH_y-plane on the S^+-line which leaves the point (p_0', H_{y_0}'). This corresponds to the case for which the first of the inequalities (4.1) to (4.4) in [3] become equalities.

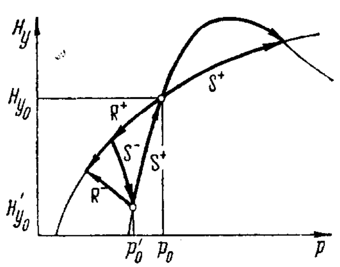

Fig. 2.

Two cases are possible:

$H_{y0}' > H_+(p_0, H_{y0}, p = p_0')$ (case 1, Fig. 1)

$H_{y0}' < H_+(p_0, H_{y0}, p = p_0')$ (case 2, Fig. 2)

In Figs. 1 and 2 in the pH_y-plane it is evident that of the two shock waves, expansion waves and contact discontinuity, the following combinations are possible:

$$R^+R^-K,\ R^+KS^-,\ S^+KS^+,\ S^-KS^- \quad \text{in case 1}$$
$$R^+KR^-,\ R^+S^-K,\ S^+KS^+,\ S^-KS^- \quad \text{in case 2}$$

The combination S^+KS^+ is possible if the S^+-line leaving the point (p_0, H_{y_0}) intersects the S^+-line leaving the point (p_0', H_{y_0}'). In the first case there may be either no point of intersection or an even number, in the second case an odd number; there will be the same number of points in the $\Delta u\Delta v$-plane, corresponding to the combination S^+KS^+. The combination S^-KS^- is possible if the corresponding equality of Section 4, [3] is fulfilled.

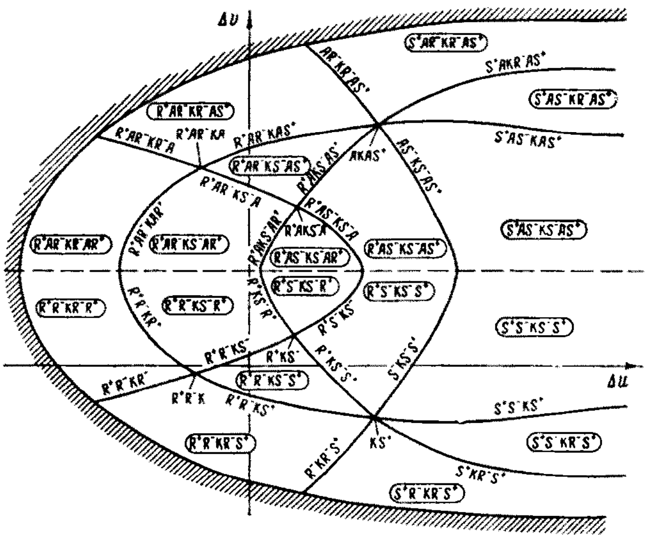

Fig. 3.

Also possible is the combination KS^- (Figs. 1,2), in which the strength of the K-discontinuity is equal to zero, if p_0 and p_0' are linked by the relations on the S^+-wave. To all these combinations there are corresponding points in the $\Delta u\Delta v$-plane (Figs. 3,4). The lines and regions in this plane are constructed as in [3]. In the combinations $R^-KR^-S^+$, $S^-KS^-S^+$, the strengths of R^-- and S^--waves, going to left and right, are equal.

If $H_{y_0} < 0$, $H_{y_0}' < 0$, then Figs. 1,2,3 and 4 do not change if $-H_y$ and $-\Delta v$ are plotted on the axis instead of H_y and Δv.

If $H_{y_0} < 0$, $H_{y_0}' > 0$, or $H_{y_0} > 0$, $H_{y_0}' < 0$, but as before,

$p_0 = p_+(p_0', |H_{y_0}'|, H_y = |H_{y_0}|)$, then in every combination occurring in a resolution, there must be an A-discontinuity, going to left or right.

In this case, the diagrams in the $\Delta u \Delta v$-plane may be qualitatively obtained from the diagrams corresponding to $H_{y_0} > 0$, $H_{y_0}' > 0$, if instead of combinations without a vortex discontinuity or with two vortex discontinuities, one puts the same combination with one vortex discontinuity,

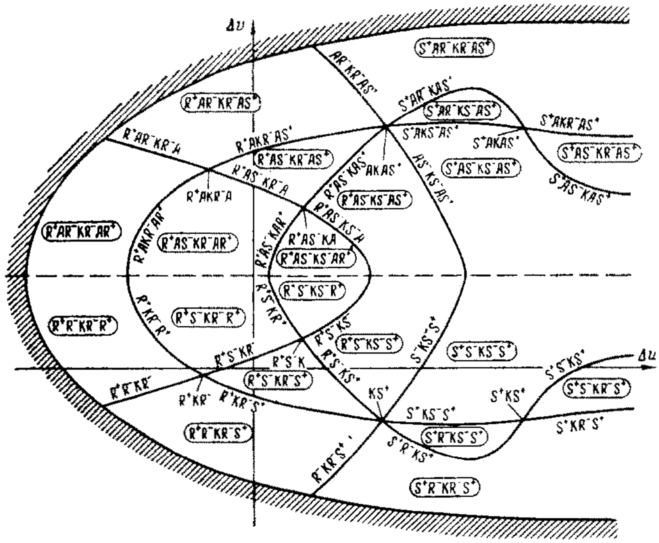

Fig. 4.

going to right or left. The criteria (2.1) to (2.4) remain the same, except that H_y must be taken to be the absolute value. The dividing line in the $\Delta u \Delta v$-plane in the given case separates the regions which correspond to combinations with a plane vortex discontinuity going to the left. It may be shown that if $H_{y_0} < 0$, $H_{y_0}' < 0$, then the regions which correspond to combinations with the vortex discontinuity going to the right lie above the dividing line, and regions which correspond to combinations with the vortex discontinuity going to the left lie below.

For example, the point which corresponds to the combination KAS^+ lies above the dividing line, the point which corresponds to the combination AKS^+ lies below. If $H_{y_0} > 0$, $H_{y_0}' < 0$, the regions which correspond to combinations with the vortex discontinuity going to the right lie below the dividing line; regions which correspond to combinations with the vortex discontinuity going to the left lie above. Thus the point which corresponds to the combination AKS^+ lies above the dividing line, the point which corresponds to the combination KAS^+ lies below.

The diagrams for the case of the resolution of an arbitrary discontinuity in which $\mathbf{H}_{\tau_0} \not\parallel \mathbf{H}_{\tau_0}'$; $p_0 = p_+(p_0', H_{y_0}', H_y = H_{y_0})$, $\Delta w \neq 0$ are obtained from the diagrams in Figs. 3 and 4 by rotating the latter about the center line. If $\mathbf{H}_{\tau_0} \not\parallel \mathbf{H}_{\tau_0}'$, then after rotation it is necessary in addition to make a translation of the resulting three-dimensional diagram with respect to the origin [3, Sect. 12].

Let us apply the case of the resolution of an arbitrary discontinuity, which has been considered, to the problem of the interaction of an S^+-wave with a K-discontinuity. This may be done, since at the moment of impact there will be a resolution of the discontinuity for which the point (p_0, H_{y_0}) of the pH_y-plane lies on the S^+-line leaving the point (p_0', H_{y_0}'). Here, $\Delta u = u_0 > 0$, $H_{y_1} = H_{y_0}' > 0$, $\Delta v = v_0 < 0$. After the impact, there may be combinations which correspond to regions lying in the $\Delta u > 0$, $\Delta v < 0$ portion of the $\Delta u \Delta v$-plane (Figs. 3,4) going in both directions. We may note that a vortex discontinuity is not created in the interaction.

The nature of the combination depends on p_0', H_{y_0}', ρ_0, ρ_1 and U, where U is the speed of the incident wave. As is evident from Figs. 3 and 4 for the corresponding value of the parameters, the S^+-wave may, on hitting the K-discontinuity, go through it, changing its strength.

Let us consider the case of the interaction of an S^+-wave with an A-discontinuity. Let the S^+-wave overtake the plane A-discontinuity moving in a gas at rest. At the moment of impact there is created a discontinuity in which $H_{y_0} < 0$, $H_{y_0}' > 0$, $\Delta u = u_0 > 0$, $\Delta v = v_0 = \phi_+ + 2h_0'V_0'$.

We note that, in the $\Delta u \Delta v$-plane, the points which correspond to the combinations KAS^+ and AKS^+ have the coordinates $\Delta u = u_0$, $\Delta v = -\phi_+' + 2h_1'V_1'$ and $\Delta u = u_0$, $\Delta v = -\phi_+' - 2|h_0|V_0$, respectively.

Consequently, the points corresponding to the combinations into which the discontinuity created by the interaction resolves itself lie on a straight line going through the points which correspond to the combinations KAS^+ and AKS^+.

In the collision of an S^+-wave with a plane A-discontinuity, $\Delta u = u_0 > 0$, $\Delta v = -\phi_+ + 2h_0'V_0$, $H_{y_0} > 0$, $H_{y_0}' < 0$ in the resulting discontinuity. The point which corresponds to the AKS^+-combination has the coordinates $\Delta u = u_0$, $\Delta v = -\phi_+' + 2h_0V_0$. Therefore, the points corresponding to the combinations into which the resulting discontinuity resolves itself lie on the straight line $\Delta u = u_0$, passing through the points which correspond to the AKS^+- and KAS^+-combinations.

2. Interaction of S^--waves with K- and A-discontinuities. Let us investigate the plane case of the resolution of an arbitrary discontinuity when the point denoting the condition (p_0, H_{y_0}) lies in the pH_y-plane on the S^--curve leaving the point (p_0', H_{y_0}') (Figs. 5, 6).

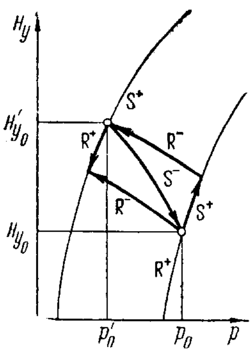

Fig. 5.

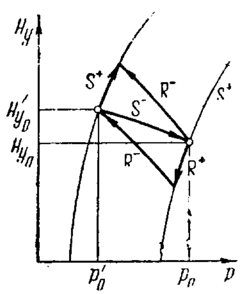

Fig. 6.

Here, the first of the inequalities (10.1) to (10.4) become equalities. Two cases are possible:

$$H_{y_0}' > H_-(p_0, H_{y_0}, p = p_0') \quad \text{(case 1, Fig. 5)}$$
$$H_{y_0}' < H_-(p_0, H_{y_0}, p = p_0') \quad \text{(case 2, Fig. 6)}$$

In Figs. 5 and 6 it is evident that of the two shock waves, expansion waves and contact discontinuity, the following combinations are possible:

S^+R^-K, R^-KR^+, S^+KS^+ S^-KS^- in case 1
R^-KS^+, R^+R^-K, S^+KS^+ S^-KS^- in case 2

The S^+KS^+-combination is possible if the S^+-line leaving the point (p_0, H_{y_0}) intersects the S^+-line leaving the point (p_0', H_{y_0}'). There may be either two points of intersection or none. The combination S^-KS^- is possible if there is an intersection of S^--lines leaving those points.

The combination KS^- is also possible (Figs. 5, 6).

For all these combinations there are corresponding points in the $\Delta u \Delta v$-plane (Figs. 7, 8). The lines and regions in this plane are constructed as in [3]. If $H_{y_0} < 0$, $H_{y_0}' < 0$; $H_{y_0} < 0$, $H_{y_0}' > 0$; $H_{y_0} > 0$,

$H_{y_0}' < 0$, then the diagrams in the $\Delta u \Delta v$-plane are altered as described in Sect. 1.

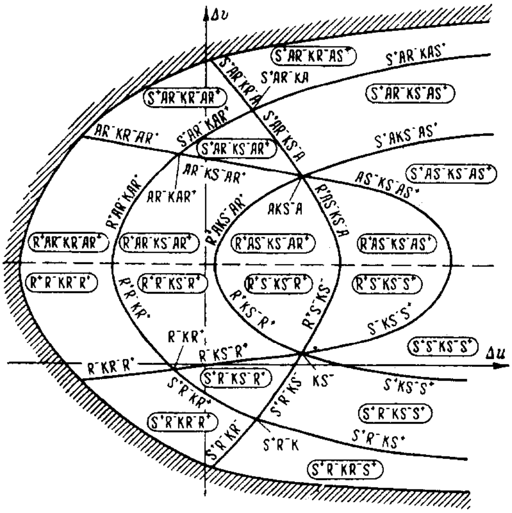

Fig. 7.

The diagrams for the case of the resolution of an arbitrary discontinuity in which $p_0 = p_-(p_0', |\mathbf{H}_{\tau_0}'|, H_y = |\mathbf{H}_{\tau_0}|), \mathbf{H}_{\tau_0} \nparallel \mathbf{H}_{\tau_0}', \Delta w \neq 0$, are obtained from the diagrams in Figs. 7 and 8 by the method described in Sect. 12 of [3].

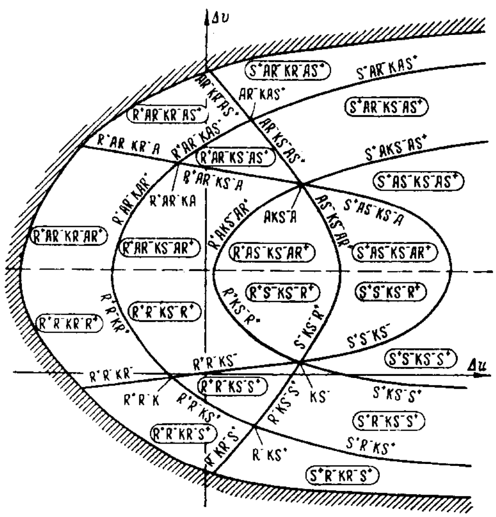

Fig. 8.

Let us investigate the interaction of S^--waves with a K-discontinuity. At the moment of impact there is created a discontinuity for which the point (p_0, H_{y_0}) of the pH_y-plane lies on the S^--line leaving the point (p_0', H_{y_0}'). On the resulting discontinuity, $\Delta u > 0$, $\Delta v = v_0 > 0$. It is easy to see that the resulting discontinuity can resolve itself into combinations which correspond to regions lying in the $\Delta u > 0$, $\Delta v > 0$ portion of the $\Delta u \Delta v$-plane, as shown in Figs. 7 and 8.

Let us investigate the interaction of an S^--wave with an A-discontinuity. Let the A-discontinuity be plane. If the A-discontinuity overtakes the S^--wave, then at impact $\Delta u = u_0 > 0$, $\Delta v = \phi_- + 2|h_0| V_0 > 0$, $H_{y_0} < 0$, $H_{y_0}' > 0$ on the resulting discontinuity; if the S^--wave collides with the A-discontinuity then $H_{y_0} > 0$, $H_{y_0}' < 0$, $\Delta u > 0$, $\Delta v = \phi_- + |2h_0'| V_0' > 0$ on the resulting discontinuity. Consequently, the resulting discontinuity resolves itself in combinations which correspond to the regions lying in the $\Delta u > 0$, $\Delta v > 0$ portion of the $\Delta u \Delta v$-plane, Figs. 7 and 8, on the straight line $\Delta u = u_0$. The straight lines $\Delta u = u_0$ pass through the points which correspond to the combinations AKS^- and KS^-A.

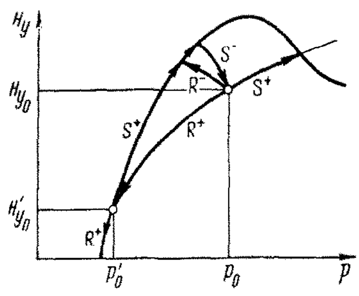

Fig. 9.

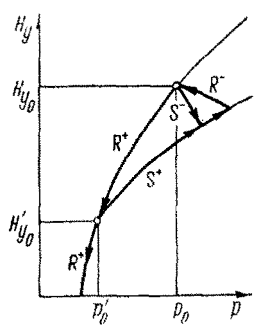

Fig. 10.

When considering the interaction of S^+-, S^--, R^+-, R^--waves with K- and A-discontinuities, it will be clear in which portion of the plane the combinations resulting from the resolution of the resulting discontinuity may lie. We note, however, that the points corresponding to the resulting discontinuity do not have an arbitrary position in the indicated portion of the plane, but only on the S^+-, S^--, R^+-, R^--lines, respectively.

3. Interaction of R^+-waves with K- and A-discontinuities. Let us consider the plane case of the resolution of an arbitrary discontinuity when the point (p_0', H_{y_0}') lies on the R^+-line leaving the point (p_0, H_{y_0}) (Figs. 9, 10). This corresponds to the case where the second of the inequalities (4.1) to (4.4) become equalities. Two cases are possible:

$$p_0 < p_+(p_0', H_{y_0}', H_y = H_{y_0}) \quad \text{(case 1, Fig. 9)}$$
$$p_0 > p_+(p_0', H_{y_0}', H_y = H_{y_0}) \quad \text{(case 2, Fig. 10).}$$

On Figs. 9 and 10 in the pH_y-plane it is evident that of the two shock waves, expansion waves and contact discontinuity, the following combinations are possible:

S^-KS^+, KR^-S^+, R^+KR^+, S^+KS^+, S^-KS^- in case 1

R^-KS^+, KS^-S^+, R^+KR^+, S^+KS^+, S^+KS^- in case 2

In case 1, the combination S^+KS^+ necessarily occurs; in case 2 it is realized if the S^+-line leaving the point (p_0', H_{y_0}') intersects the S^+-line leaving the point (p_0, H_{y_0}). The S^-KS^--combination is realized if the S^--line leaving the point (p_0', H_{y_0}') intersects the S^--line leaving the point (p_0, H_{y_0}). The combination R^+K is also possible (Figs. 9,10). The R^+KR^+-combination corresponds to a line in the $\Delta u\Delta v$-plane. All the remaining combinations correspond to lines in the $\Delta u\Delta v$-plane (Figs. 11, 12).

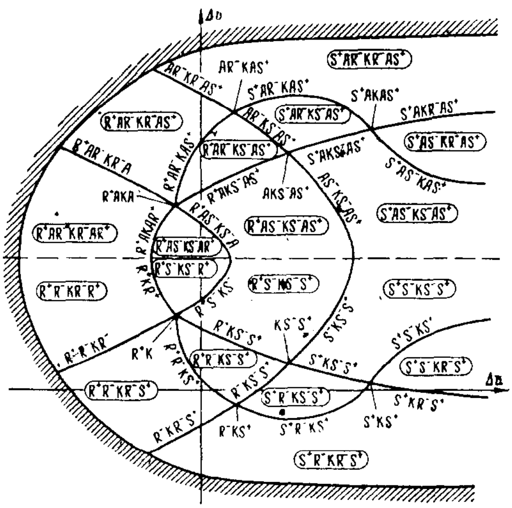

Fig. 11.

Lines and regions in the $\Delta u\Delta v$-plane are constructed in the same way as in the preceding cases. If $H_{y_0} < 0$, $H_{y_0}' < 0$; $H_{y_0} > 0$, $H_{y_0}' < 0$, etc., then the diagrams in the $\Delta u \Delta v$ -plane will be altered as described in Sect. 1.

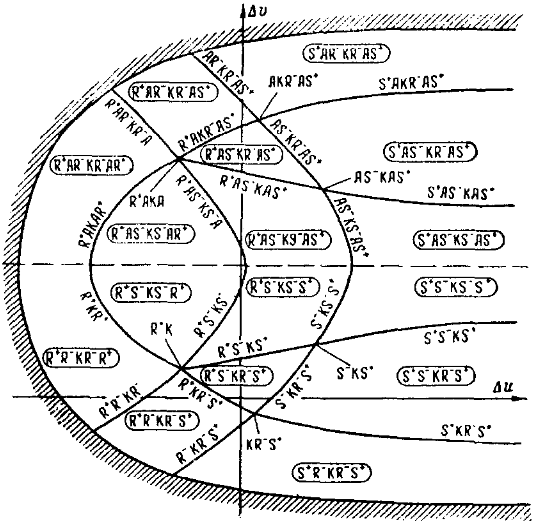

Fig. 12.

The diagrams for the case of the resolution of an arbitrary discontinuity for which $|\mathbf{H}_{\tau_0}'| = H_+(p_0, |\mathbf{H}_{\tau_0}|, p = p_0')$, $\mathbf{H}_{\tau_0} \not\parallel \mathbf{H}_{\tau_0}'$, $\Delta w \neq 0$ are obtained from the diagrams in Figs. 11 and 12 by the method described in Sect. 12 of [3].

Let us consider the interaction of an R^+-wave with a K-discontinuity. Recall that in the case of interaction of R^+-, R^--waves with each other and with other waves and discontinuities we will be interested in the combinations of waves into which the interactions under consideration resolve themselves. It is assumed, as before, that the interpenetration is completed in a finite time.

In the discontinuity created, $\Delta u < 0$, $\Delta v > 0$. The result of the interaction will be combinations of waves going in both directions, which correspond to regions lying in the $\Delta u < 0$, $\Delta v > 0$ portion of the $\Delta u \Delta v$-plane (Figs. 11, 12).

Let us consider the interaction of an R^+-wave with a plane A-discontinuity. On the resulting discontinuity $\Delta u = u_0 < 0$, $\Delta v = -\chi_+ + 2h_0 V_0$, $H_{y_0} < 0$, $H_{y_0}' > 0$, when the R^+-wave overtakes the A-discontinuity, and $\Delta u = u_0 < 0$, $\Delta v = \chi_+ + 2|h_0'| V_0$, $H_{y_0} > 0$, $H_{y_0}' < 0$ when the R^+-wave collides with the A-discontinuity. The possible combinations correspond to points lying on the straight line $\Delta u = u_0$.

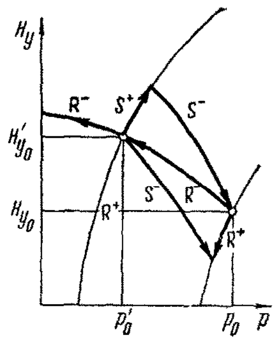

Fig. 13.

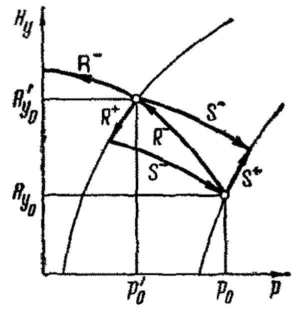

Fig. 14.

In both cases the straight lines pass through the points which correspond to the combinations R^+KA and R^+AK.

4. Interaction of R^--waves with K- and A-discontinuities. Let us consider the plane case of the resolution of an arbitrary discontinuity when the point (p_0', H_{y_0}') of the pH_y-plane lies on the R^--line leaving the point (p_0, H_{y_0}) (Figs. 13, 14). This corresponds to the case where the second of the inequalities (10.1) to (10.4) become equalities. Two cases are possible

$$p_0 > p_-(p_0', H_{y_0}' H_y = H_{y_0}) \qquad \text{(case 2, Fig. 14)}$$
$$p_0 < p_-(p_0', H_{y_0}' H_y = H_{y_0}) \qquad \text{(case 1, Fig. 13)}$$

On Figs. 13 and 14 in the pH_y-plane it is evident that of the two shock waves, expansion waves and contact discontinuity, the following combinations are possible:

KS^-S^+, R^+KS^-, R^-KR^-, S^-KS^-, S^+KS^+ in case 1

KS^-R^+, S^+KS^-, R^-KR^-, S^-KS^-, S^+KS^+ in case 2

The combination S^+KS^+ is possible if the S^+-lines leaving the points (p_0, H_{y_0}), (p_0', H_{y_0}') intersect each other. The combination S^-KS^- is realized if the S^--lines leaving those points intersect each other. Also possible is the combination R^-K (Figs. 13, 14). The combination R^-KR^- corresponds to a line in the $\Delta u \Delta v$-plane. The lines and regions in the $\Delta u \Delta v$-plane (Figs. 15, 16) are constructed as in [3]. If the inequalities $H_{y_0} > 0$, $H_{y_0}' > 0$ change sign, then Figs. 13-16 will be altered as described in Sect. 1.

The diagrams for the case of the resolution of an arbitrary discontinuity for which $|\mathbf{H}_{\tau_0}'| = H_-(p_0, |\mathbf{H}_{\tau_0}|, p = p_0')$, $\mathbf{H}_{\tau_0} \nparallel \mathbf{H}_{\tau_0}'$, $\Delta w \neq 0$ are obtained from the diagrams in Figs. 15 and 16 by the method described in Sect. 12 of [3].

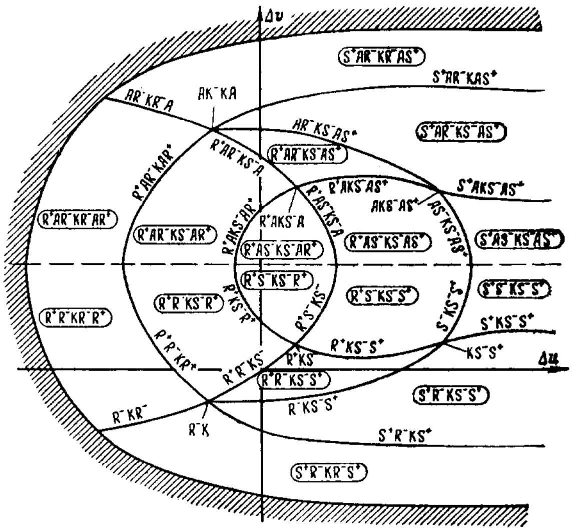

Fig. 15.

The interaction of R^--waves with each other and with K- and A-discontinuities is reduced to such a case of the resolution of an arbitrary discontinuity. Let us consider the interaction of R^--waves with a K-discontinuity. In the discontinuity which results from the interaction, $\Delta u < 0$, $\Delta v < 0$. The result of the interactions will be combinations which correspond to regions lying in the portion $\Delta u < 0$, $\Delta v < 0$ of the $\Delta u \Delta v$-plane (Figs. 15, 16).

In the interaction of an R^--wave with an A-discontinuity, the possible combinations correspond to points lying on the straight lines $\Delta u = u_0$, passing through the points which correspond to the combinations AR^-K and R^-KA.

Above, we considered the interactions of S^+-, S^--, R^+-, R^--waves with a plane A-discontinuity. In the interaction of these waves with a three-dimensional A-discontinuity, there results a discontinuity for which $\mathbf{H}_{\tau_0} \nparallel \mathbf{H}_{\tau_0}'$; $\Delta w \neq 0$. To investigate this, it is necessary to use the three-dimensional diagrams which are obtained from the diagrams of Figs. 3, 4, 7, 8, 11, 12, 15 and 16 by the method described in Sect. 12 of [3]. The aggregate of points which correspond to combinations into which the resulting discontinuity may resolve itself lie in the planes $\Delta u = u_0$ passing through the points which correspond to the combinations KS^+, KS^-, R^+K, R^-K.

If the lines S^+, R^+ (Figs. 1, 2, 9, 10) intersect, then the pictures in the $\Delta u \Delta v$-plane differ from Figs. 3, 4, 11 and 12 in that the lines $R^+KS^-S^+$ and $R^+R^-KS^+$ in Figs. 3 and 12, and $R^+S^-KS^+$ and $R^+KR^-S^+$ in Figs. 4 and 11, intersect each other at points which correspond to the combination R^+KS^+. In the first case, the lines $R^+S^-KS^+$ and $S^+KR^-S^+$ leave this point, and in the second case the lines $R^+KS^-S^+$ and $R^+R^-KS^+$; these intersect at the points KS^+ and R^+K, respectively. If the lines S^-R^- (Figs. 5, 6, 13, 14) intersect, then the pictures in the $\Delta u \Delta v$-plane differ from Figs. 7, 8, 15 and 16 in that the lines $R^-KS^-R^+$, $S^+R^-KS^-$ of Figs. 7 and 15, and $R^+R^-KS^-$, $R^-KS^-S^+$ of Figs. 8 and 16, intersect at points which correspond to the combination R^-KS^-. In the first case, the lines $R^+R^-KS^-$ and $R^-KS^-S^+$ leave this point, and in the second case the lines $R^-KS^-R^+$ and $S^+R^-KS^-$; these intersect at the points KS^- and R^-K, respectively.

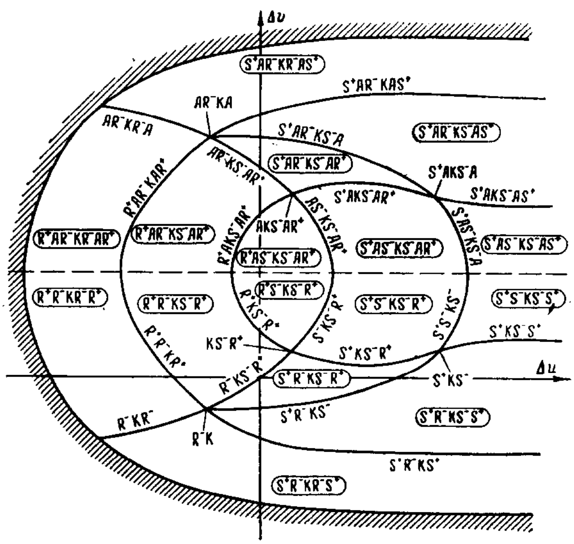

Fig. 16.

We note that if, in the cases described by inequalities (2.1), (2.3) of [3], the lines S^+ and R^+, and in the cases described by inequalities (10.1) and (10.3), the lines S^+ and R^+ leaving points (p_0', H_{y_0}'), (p_0, H_{y_0}), have two intersections, then in the $\Delta u \Delta v$-plane the lines joining the points (R^+R^-K) (R^-KS^+) and (R^+KS^-) (KS^-S^+) in Fig. 7; (R^+KR^-) (KR^-S^+) and (R^+S^-K) (S^-KS^+) in Fig. 9; (R^+R^-K) (R^+KS^-) and

(R^-KS^+) (KS^-S^+) in Fig. 15; (R^-KR^+) (KS^-R^+) and (S^+R^-K) (S^+KS^-) in Fig. 17, have two intersections. In Figs. 7 and 9 the points of intersection correspond to the combination R^+KS^+, in Figs. 15 and 17 to the combination R^-KS^-.

We note also that if $\Delta w \neq 0$, $\mathbf{H}_{\tau_0} \nparallel \mathbf{H}_{\tau_0}'$ on the discontinuity, the rotation of the figure should not be around the dividing line, but about the center line, which does not coincide everywhere with the dividing line (as was incorrectly proved in [3]), but only at some points.

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